# How strong is the natural greenhouse-gas effect?

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The observed global mean earth surface temperature $T_g$ is about $15°C$. A very simple radiation balance model which excludes the atmosphere has an equilibrium solution $T_g^* = -18°C$. The difference of $33°C$ is sometimes attributed to the natural greenhouse-gas effect. This view is conceptionally wrong. The difference originates from the presence of the atmosphere with its effects on the short-wave and long-wave radiation as well as other energy fluxes like convection. We first calculate the surface temperature of a planet without atmosphere. Then we introduce a highly parameterized atmosphere and estimate surface equilibrium temperatures under different conditions. Finally we estimate and compare sensitivities of the surface temperature with respect to changes in the energy fluxes.

## 1 The Earth without atmosphere

The energy balance of a planet without atmosphere is pretty easy. Only two fluxes need to be considered: The shortwave radiation uptake S and the longwave outgoing radiation L from the surface. According to the 1st law of thermodynamics, changes in the surface temperature are directly proportional to changes of the energy at the surface with the heat capacity c as constant of proportionality

$$c\frac{dT_g}{dt} = S - L. \tag{1}$$

The surface reflects part of the incoming radiation $So$. The reflected fraction of radiation is called albedo $\alpha$. The radiation uptake is therefore

$$S = So(1 - \alpha). \tag{2}$$

The outgoing radiation $L$ can be expressed by the Stefan-Boltzmann law as

$$L = \sigma\varepsilon Tg^4 \tag{3}$$

with the Stefan-Boltzmann constant $\sigma$ and the emissivity $\varepsilon$. Finally we have to consider that only the area of a circle absorbs solar energy while the whole surface of the sphere emits longwave radiation. Since the area of a circle with the same radius as a sphere is only 1/4 of the area of the sphere we can write $So = Io/4$ and get

$$c\frac{dT_g}{dt} = \frac{Io}{4}(1-\alpha_g) - \sigma\varepsilon T_g^4. \tag{4}$$

The equilibrium surface temperature therefore is

$$T_g = \left(\frac{Io(1-\alpha_g)}{4\sigma\epsilon}\right)^{1/4} \tag{5}$$

And with realistic values for the Earth of $\alpha_g = 0.3$, $\varepsilon = 1$ and $Io = 1370W/m^2$ we get $T_g = 255K = -18°C$ as the only real and positive solution. The parameter $\alpha_g = 0.3$ is the observed planetary albedo, i.e. the real Earth reflects 30% of the solar radiation. However, the real Earth has an atmosphere. The albedo of the Earth surface is about 0.1, i.e. the surface reflects only 10% of the incoming solar radiation. If we use this value for $\alpha_g$ we get $271K = -2°C$ which is only $17°C$ below the observed surface temperature on Earth.

# 2 The Earth with atmosphere

The atmosphere interacts with shortwave and longwave radiation. The radiation is separated in 3 fractions. Part of the radiation is transmitted, part is absorbed and part is reflected. We use the letters $t, a, r$ for the fractions, respectively, and apply this to both, shortwave and longwave radiation. Furthermore we have a convective energy flux $C$ from the ground surface into the atmosphere and we allow for a heating of the surface due to an anthropogenic greenhouse-gas effect $A_{GHG} = \Delta r_l \varepsilon\sigma T_g^4$. The ground surface energy balance can now be written as

$$c_g\frac{dT_g}{dt} = (1-\alpha_g)t_s S - (1-r_l)\varepsilon\sigma T_g^4 - C + A_{GHG} \tag{6}$$

and the equilibrium temperature follows as

$$T_g^* = \left(\frac{(1-\alpha_g)t_s S - C + A_{GHG}}{(1-r_l)\varepsilon\sigma}\right)^{1/4.} \tag{7}$$

If we neglect the whole atmosphere in a first step we get the same solution as in the previous section,

$$T_g^*(t_s = 1, C = 0, A_{GHG} = 0, r_l = 0) = 271K = -2°C. \tag{8}$$

Now what would the surface temperature be if only the natural greenhouse-gas effect would work? Observations in the atmosphere show that $r_l$ is about 96/114 which means that about 84.2% of the radiation emitted from the ground is reflected by the atmosphere. In this case we get

$$T_g^*(t_s = 1, C = 0, A_{GHG} = 0, r_l = 0.842) = 431K = 158°C. \tag{9}$$

If we consider only the atmospheric effect on the shortwave radiation, which in case of the Earth transmits about 50% of the incoming solar radiation, we get

$$T_g^*(t_s = 0.5, C = 0, A_{GHG} = 0, r_l = 0) = 228K = -45°C. \quad (10)$$

If we consider both radiative terms we get

$$T_g^*(t_s = 0.5, C = 0, A_{GHG} = 0, r_l = 0.842) = 362K = 89°C. \quad (11)$$

This means that the pure radiative equilibrium temperature at the Earth surface would be nearly $90°C$.

If we consider the observed convective energy flux from the Earth surface into the atmosphere, which is about $0.27 So = 92.4W/m^2$ we get

$$T_g^*(t_s = 0.5, C = -92.4W/m^2, A_{GHG} = 0, r_l = 0.842) = 288.068K = 15°C. \quad (12)$$

This shows that the observed surface temperature depends strongly on convective cooling. The temperature of the earth without atmosphere would be $271K$. The natural greenhouse-gas effect alone would heat to $431K$, i.e. by $160K$. The radiative cooling due to the atmospheric transmissivity of solar radiation would cool to $228K$, i.e. by $43K$. This cannot compensate for the greenhouse effect and the radiation temperature would be $362K$. The observed surface temperature can only be reproduced if the strong energy flux of convective cooling is considered which cools the surface by about $74K$..

In a last step of this section we ask how the surface temperature would be if there was an anthropogenic greenhouse-gas effect of $2W/m2$.

$$T_g^*(ts = 0.5, C = -92.4W/m^2, A_{GHG} = 2, rl = 0.842) = 290.377K = 17°C. \quad (13)$$

An anthropogenic greenhouse-gas effect of $2W/m^2$ would heat the surface equilibrium temperature by $2.31K$.

We see that in none of the discussed cases a surface temperature of $255K$ is realized. The surface temperature without changes of the surface albedo would be $-2°C$. The real Earth is partly covered by water. And at least part of it would freeze under these conditions and enhance the surface albedo which would lead to lower temperature and more ice. Thus the Earth would further cool down by this ice-albedo feedback. Since the surface temperature without convective cooling is $362K$ while it is $288K$ if convection is considered we can conclude that convection cools the surface by $74K$.

All the calculations neglect feedback mechanism (like the ice-albedo feedback) as they occur on the real Earth. We just compared different energy fluxes and how they contribute to the observed temperature on Earth. We learn that the surface temperature stems from a balance of large energy fluxes which allow for a wide range of temperatures if one of them would be switched of. The next section discusses the sensitivity of the equilibrium surface temperature with respect to the energy fluxes.

## 3 Climate Sensitivity

The climate sensitivity $\lambda$ is the change of the equilibrium surface temperature with changes in forcing or internal parameters. Given our basic model equation

$$T_g^* = \left( \frac{(1-\alpha_g)t_s S - C + A_{GHG}}{(1-r_l)\varepsilon\sigma} \right)^{1/4} \tag{14}$$

we build the following climate sensitivities:

$$\begin{aligned}
\lambda_S &= \frac{\partial T_g^*}{\partial S} &= \frac{(1-\alpha_g)ts}{4T_g^3(1-r_l)\varepsilon\sigma} = 0.52\frac{K}{Wm^2} \\
\lambda_{t_s} &= \frac{\partial T_g^*}{\partial t_s} &= \frac{(1-\alpha_g)S}{4T_g^3(1-r_l)\varepsilon\sigma} = 3.6\frac{K}{\%} \\
\lambda_C &= \frac{\partial T_g^*}{\partial C} &= \frac{-1}{4T_g^3(1-r_l)\varepsilon\sigma} = -1.168\frac{K}{Wm^2} \\
\lambda_{GHG} &= \frac{\partial T_g^*}{\partial A_{GHG}} &= -\frac{dT_g}{dC} \\
\lambda_{r_l} &= \frac{\partial T_g^*}{\partial r_l} &= \frac{(1-\alpha_g)t_s S - C + A}{4T_g^3(1-r_l)^2\varepsilon\sigma} = 4.56\frac{K}{\%}
\end{aligned} \tag{15}$$

We see that the sensitivity of surface temperature with respect to solar radiation is about $.5K/(W/m^2)$. The sensitivity with respect to the internal processes, convection and greenhouse gases, are the same but with opposite sign. They are about twice as large as the sensitivity with respect to solar radiation. The sensitivity with respect to $t_s$ and $r_l$ are very high. A one percent change in transmissivity of the atmosphere causes a surface temperature increase of $3.6K$. A temperature increase of $4.56K$ results from an increase of 1% in the reflectivity of the atmosphere with respect to longwave radiation.

Though we regarded a very simple model here we could learn a lot about equilibrium temperature, the magnitudes of different mechanisms that drive surface temperature and the sensitivity of equilibrium surface temperature with respect to changes in forcings.

The model is highly parameterized but in good agreement with more explicit radiation models (Moeller and Manabe, 1961) and radiative-convective models (Augustsson and Ramanathan, 1977).

## References

Augustsson, T. and V. Ramanthan, 1977: A Radiative-Convective Model Study of the CO2 Climate Problem. J. Atm. Sci., 34, 448-451.

Moeller, F. and S. Manabe, 1961: Ueber das Strahlungsgleichgewicht der Atmosphaere. Zeitschrift fuer Meteorologie, 15, 3-8.